Spectral Domain Solution of Arbitrary Coplanar Transmission Line with Multilayer Substrate

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***Abstract*—A hybrid mode analysis is presented for a multilayer dielectric within a rectangular conducting box. An arbitrary set of conductors may be distributed along the lower surface of the top layer, so that single or coupled forms may be analyzed of slot line, microstrip, or coplanar waveguide. The analysis combines a transfer-matrix approach with the spectral domain method to give a versatile and efficient solution. CPU time on an IBM 360/65 is about 1 s per layer of substrate, for a single slot or strip, at one frequency.**

Introduction

With the increased use of suspended substrate lines (such as slot line and coplanar waveguide), various studies have been made of multilayer "microstrip-type" structures. Generally, these have been limited to the quasi-TEM approximation, with consequent restriction to dispersion-free operation [1], [2]. In this short paper, the spectral domain method of Denlinger [3] and Itoh and Mittra [4] is applied to give a frequency-dependent hybrid mode analysis of an enclosed multilayer structure.

The method uses a transfer-matrix approach in the spectral domain and gives advantages of flexibility and efficiency. Due to this approach, a single computer program can analyze single or coupled strips and single or coupled slots. Simple program modifications would treat an arbitrary distribution of coplanar conductors. The conductors are on the top surface of an arbitrary number of dielectric layers. The aforementioned structure is, then, with one dielectric region above the coplanar conductors, enclosed within rectangular conducting walls. In its generality, it naturally includes the particular structures studied by Itoh and Mittra [4] with $N = 1$ and by Knorr and Kuchler [5] with $N = 2$, the latter taking no account of an enclosure. The transfer-matrix approach makes for easy setting up of the equations and computer program, and gives computing time at worst linear in N. It also gives directly the fields at any plane of interest below the coplanar conductors.

A brief account will now be given of the theory, followed by examples of results for single microstrip and for single and coupled slots on a suspended substrate.

Theory

Consider a multilayer structure with distributed conductors on the top of the Nth layer (Fig. 1). To obtain the dispersion characteristics of this type of structure, the "spectral domain" method is employed [4]. Let the electrical potential function be $\psi^e(x,y)$ and the magnetic potential function be $\psi^h(x,y)$. In this case, the z components of fields are

$$E_z = j\frac{k^2-\beta^2}{\beta}\psi^e(x,y)e^{-j\beta z}$$

$$H_z = j\frac{k^2-\beta^2}{\beta}\psi^h(x,y)e^{-j\beta z}. \quad (1)$$

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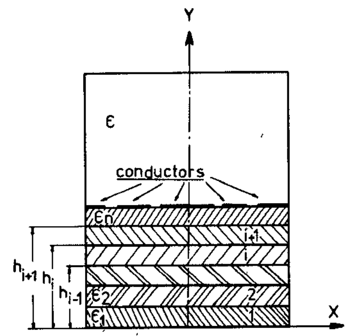

Fig. 1. Shielded multilayer dielectric with arbitrary coplanar conductors.

The potential functions can be written in the Fourier domain as

$$\tilde{\psi}^e(\alpha_n, y) = \int_{-a}^{+a} \psi^e(x,y)e^{-j\alpha_n x}\,dx$$

$$\tilde{\psi}^h(\alpha_n, y) = \int_{-a}^{+a} \psi^h(x,y)e^{-j\alpha_n x}\,dx. \quad (2)$$

From the preceding functions, all fields can be expressed in the Fourier domain. For the sake of simplicity, transformation of any field is shown by a tilde (~), for example, the transformation of E_x is shown as $\tilde{E}_x$. Now consider two successive layers, say, i and $i+1$. Potential functions within these two layers and in the Fourier domain are

$$\tilde{\psi}_i^e(\alpha_n,y) = A_i sh\gamma_{in}y + B_i ch\gamma_{in}y,$$

$$\tilde{\psi}_i^h(\alpha_n,y) = C_i sh\gamma_{in}y + D_i ch\gamma_{in}y, \qquad h_{i-1} < y < h_i \quad (3)$$

$$\tilde{\psi}_{i+1}^e(\alpha_n,y) = A_{i+1} sh\gamma_{(i+1)n}y + B_{i+1} ch\gamma_{(i+1)n}y,$$

$$\tilde{\psi}_{i+1}^h(\alpha_n,y) = C_{i+1} sh\gamma_{(i+1)n}y + D_{i+1} ch\gamma_{(i+1)n}y,$$

$$h_i < y < h_{i+1} \quad (4)$$

where

$$\gamma_{in}^2 = \alpha_n^2 + \beta^2 - K_i^2.$$

Boundary conditions at the interface of the ith layer and $i+1$th layer are

$$\tilde{E}_{zi} = \tilde{E}_{z(i+1)},$$

$$\tilde{H}_{zi} = \tilde{H}_{z(i+1)},$$

$$\tilde{E}_{xi} = \tilde{E}_{x(i+1)},$$

$$\tilde{H}_{xi} = \tilde{H}_{x(i+1)}, \qquad \text{at } y = h_i. \quad (5)$$

Substituting appropriate fields in the preceding boundary conditions yields a relation between $A_i, B_i, C_i, D_i, A_{i+1}, B_{i+1}, C_{i+1}, D_{i+1}$, that is,

$$\begin{bmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \end{bmatrix} = [\gamma_{i+1}h_i]^{-1}\cdot[\gamma_i h_i]\cdot \begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix} \quad (6)$$

where

$$[\gamma_i h_i] = \begin{bmatrix} (K_i^2 - \beta^2) sh\gamma_i h_i & (K_i^2 - \beta^2) ch\gamma_i h_i & 0 & 0 \\ 0 & 0 & (K_i^2 - \beta^2) sh\gamma_i h_i & (K_i^2 - \beta^2) ch\gamma_i h_i \\ -j\alpha_n sh\gamma_i h_i & -j\alpha_n ch\gamma_i h_i & \frac{\omega\mu_i\gamma_i}{\beta} ch\gamma_i h_i & \frac{\omega\mu_i h_i}{\beta} sh\gamma_i h_i \\ -\frac{\omega\varepsilon_i\gamma_i}{\beta} ch\gamma_i h_i & -\frac{\omega\varepsilon_i\gamma_i}{\beta} sh\gamma_i h_i & -j\alpha_n sh\gamma_i h_i & -j\alpha_n ch\gamma_i h_i \end{bmatrix} \tag{7}$$

$$[\gamma_{i+1} h_i]^{-1} = \frac{-1}{K_{i+1}^2 - \beta^2} \cdot \begin{bmatrix} sh\gamma_{i+1} h_i & j\frac{\alpha_n\beta}{\omega\varepsilon_{i+1}\gamma_{i+1}} ch\gamma_{i+1} h_i & 0 & \frac{\beta(K_{i+1}^2 - \beta^2)}{\omega\gamma_{i+1}\varepsilon_{i+1}} ch\gamma_{i+1} h \\ -ch\gamma_{i+1} h_i & -j\frac{\alpha_n\beta}{\omega\varepsilon_{i+1}\gamma_{i+1}} sh\gamma_{i+1} h_i & 0 & -\frac{\beta(K_{i+1}^2 - \beta^2)}{\omega\gamma_{i+1}\varepsilon_{i+1}} sh\gamma_{i+1} h_i \\ -j\frac{\alpha_n\beta}{\omega\gamma_{i+1}\mu_{i+1}} ch\gamma_{i+1} h_i & sh\gamma_{i+1} h_i & -\frac{\beta(K_{i+1}^2 - \beta^2)}{\omega\gamma_{i+1}\mu_{i+1}} ch\gamma_{i+1} h_i & 0 \\ -j\frac{\alpha_n\beta}{\omega\gamma_{i+1}\mu_{i+1}} sh\gamma_{i+1} h_i & -ch\gamma_{i+1} h_i & \frac{\beta(K_{i+1}^2 - \beta^2)}{\omega\gamma_{i+1}\mu_{i+1}} sh\gamma_{i+1} h_i & 0 \end{bmatrix}. \tag{8}$$

The 4 × 4 matrix of (6) can be called a "transfer" or "chain" matrix as coefficients of the fields of any layer can be expressed in terms of coefficients of any other one, for instance, coefficients of fields of the $i + 2$th layer in terms of the ith layer are

$$\begin{bmatrix} A_{i+2} \\ B_{i+2} \\ C_{i+2} \\ D_{i+2} \end{bmatrix} = [\gamma_{i+2} h_{i+1}]^{-1} \cdot [\gamma_{i+1} h_{i+1}] \cdot [\gamma_{i+1} h_i]^{-1} \cdot [\gamma_i h_i] \cdot \begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix}. \tag{9}$$

Hence (6) is quite general and like (9) can be extended for N-layer structures. Another advantage of (6) is its simplicity and efficiency in computer program. Computing time would not increase more than linearly with the number of layers. By contrast, the method as described by Farrar and Adams [6] would give computing time proportional to N^3 for matrix "inversion."

The strips on the top layer cause a new boundary condition which can be straightforwardly satisfied for an individual problem via the method explained in [3].

This general theory can be applied to the various types of structures. One of them is shielded slot line with suspended dielectric (Fig. 2). In this special case, there are only two layers under the coplanar conductors; at the bottom of the lower one is a perfect conductor and on the top of the second one is a single slot. To satisfy the boundary conditions at the bottom, B_1 and C_1 must be zero. Hence the transfer matrix for this type of structure gives

$$\begin{bmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{bmatrix} = [\gamma_2 h_1]^{-1} [\gamma_1 h_1] \begin{bmatrix} A_1 \\ 0 \\ 0 \\ D_1 \end{bmatrix}. \tag{10}$$

It is apparent that the field coefficients in the hatched region of Fig. 2 can be expressed in terms of A_1 and D_1.

To satisfy boundary conditions at the interface of the second layer and upper air region, fields in the latter region should be evaluated. These field components are readily obtainable from

$$\tilde{E}_{z0} = j\frac{K_0^2 - \beta^2}{\beta} A_0 sh\gamma_{0n}(h_2 + d - y)$$

$$\tilde{H}_{z0} = j\frac{K_0^2 - \beta^2}{\beta} B_0 sh\gamma_{0n}(h_2 + d - y) \tag{11}$$

where

$$\gamma_{0n}^2 = \alpha_n^2 + \beta^2 - K_0^2.$$

Therefore, for matching the fields at the interface of the dielectric and upper air region, the following boundary conditions are used:

$$\tilde{E}_{z2} = \tilde{E}_{z0},$$

$$\tilde{E}_{x2} = \tilde{E}_{x0},$$

$$\tilde{H}_{x2} - \tilde{H}_{x0} = \tilde{J}_z,$$

$$\tilde{H}_{z2} - \tilde{H}_{z0} = \tilde{J}_x, \quad \text{at } y = h_2 \tag{12}$$

where $\tilde{J}_x$ and $\tilde{J}_z$ are Fourier transformations of the transverse and longitudinal currents, respectively. From (12), A_1, D_1, A_0, and B_0 are obtained. Hence $\tilde{E}_z$ and $\tilde{E}_x$ at the mentioned interface can be expressed in terms of J_x and J_z, that is,

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix} = \begin{bmatrix} \tilde{E}_z \\ \tilde{E}_x \end{bmatrix} \tag{13}$$

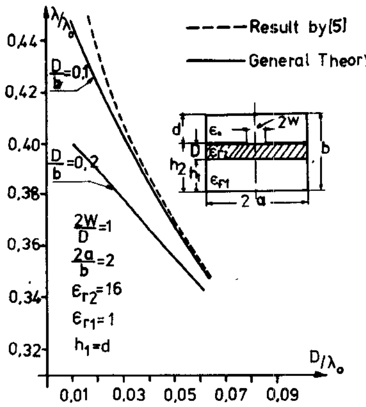

Fig. 2. Dispersion characteristic of shielded slot on a suspended substrate.

where

$$G_{11} = j\frac{K_0^2 - \beta^2}{\beta} A_{MI} sh\gamma_{0n}d$$

$$G_{12} = j\frac{K_0^2 - \beta^2}{\beta} A_{NI} sh\gamma_{0n}d$$

$$G_{21} = -j\left(\alpha_n A_{MI} + \frac{\omega\mu_0\gamma_{0n}}{\beta} A_{M2}\right) sh\gamma_{0n}d$$

$$G_{22} = -j\left(\alpha_n A_{NI} + \frac{\omega\mu_0\gamma_{0n}}{\beta} A_{N2}\right) sh\gamma_{0n}d$$

$$A_{MI} = (a''h''d_0 - h_0f''a'' + b''d_0g'' + h_0 1''b'')/\det$$

$$A_{NI} = (a''b_0f'' - a''d''d_0 - 1''b_0b'' - b''d_0C'')/\det$$

$$A_{M2} = (1''h''a_0 + a_0f''g'' + 1_0 1''b'' - a''h''C_0 - C_0g''b'' - a''f''1_0)/\det$$

$$A_{N2} = (a''d''C_0 + C_0C''b'' - 1''d''a_0 - C''f''a_0)/\det$$

$$\det = a''C_0(h_0d'' - h''b_0) + a''1_0(d''d_0 - f''b_0) + b''C_0(C''h_0 - b_0g'') + b''1_0(1''b_0 + d_0C'') + a_0C''(h''d_0 - h_0f'') + a_0g''(f''b_0 - d_0d'') + a_0 1''(h''b_0 - d''h_0)$$

$$a_0 = \frac{K_0^2 - \beta^2}{\beta} sh\gamma_{0n}d$$

$$b_0 = \frac{K_0^2 - \beta^2}{\beta} ch\gamma_{0n}d$$

$$\gamma_{0n}^2 = \alpha_n^2 + \beta^2 - K_0^2$$

$$C_0 = -\alpha_n sh\gamma_{0n}d$$

$$d_0 = -\frac{\omega\varepsilon_0\gamma_{0n}}{\alpha} sh\gamma_{0n}d$$

$$1_0 = \frac{\omega\varepsilon_0\gamma_{0n}}{\beta} ch\gamma_{0n}d$$

$$h_0 = -\alpha_n ch\gamma_{0n}d$$

$$a'' = \frac{1}{\beta}\left[(K_1^2 - \beta^2)ch\gamma_{2n}(h_2 - h_1) + \frac{\varepsilon_1\gamma_{1n}}{\varepsilon_2\gamma_{2n}}(K_2^2 - \beta^2)Sh\gamma_{2n}(h_2 - h_1)Ch\gamma_{1n}h_1\right]$$

$$b'' = \frac{1}{\beta}\left[\frac{\alpha_n\beta}{\omega\varepsilon_2\gamma_{2n}}(K_1^2 - K_2^2)sh\gamma_{2n}(h_2 - h_1)ch\gamma_{1n}h_1\right]$$

$$C'' = -\frac{1}{\beta}\frac{\alpha_n\beta}{\omega\gamma_{2n}\mu_2} \cdot [(K_1^2 - K_2^2)sh\gamma_{2n}(h_2 - h_1)sh\gamma_{1n}h_1]$$

$$d'' = \frac{1}{\beta}\left[(K_1^2 - \beta^2)ch\gamma_{2n}(h_2 - h_1)ch\gamma_{1n}h_1 + \frac{\mu_1\gamma_{1n}}{\mu_2\gamma_{2n}}(K_2^2 - \beta^2)sh\gamma_{2n}(h_2 - h_1)sh\gamma_{1n}h_1\right]$$

$$1'' = -\alpha_n\left[ch\gamma_{2n}(h_2 - h_1)sh\gamma_{1n}h_1 + \frac{\varepsilon_1\gamma_{1n}}{\varepsilon_2\gamma_{2n}} sh\gamma_{2n}(h_2 - h_1)ch\gamma_{1n}h_1\right]$$

$$f'' = -\frac{1}{\omega\varepsilon_2\gamma_{2n}\beta}\left[(K_1^2K_2^2 - \beta^2K_2^2 - \alpha_n^2K_1^2) \cdot sh\gamma_{2n}(h_2 - h_1)ch\gamma_{1n}h_1 - K_2^2\gamma_{2n}^2 \cdot \frac{\mu_1\gamma_{1n}}{\mu_2\gamma_{2n}} ch\gamma_{2n}(h_2 - h_1)sh\gamma_{1n}h_1\right]$$

$$g'' = \frac{1}{\omega\gamma_{2n}\mu_2\beta}\left[(K1^1K2^2 - \beta^2K_2^2 - \alpha_n^2K_1^2) \cdot sh(\gamma_{2n}(h_2 - h_1))sh(\gamma_{1n}h_1) - K_2^2\gamma_{2n}^2 \cdot \frac{\varepsilon_1\gamma_{1n}}{\varepsilon_2\gamma_{2n}} ch(\gamma_{2n}(h_2 - h_1))ch(\gamma_{1n}h_1)\right]$$

$$h'' = -\alpha_n\left[ch(\gamma_{2n}(h_2 - h_1))ch(\gamma_{1n}h_1) + \frac{\mu_1\gamma_{1n}}{\mu_2\gamma_{2n}} \cdot sh(\gamma_{2n}(h_2 - h_1))sh(\gamma_{1n}h_1)\right].$$

Since the fields within the slot region may be more accurately approximated, it is then appropriate to use the following form of (13):

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{E}_z \\ \tilde{E}_x \end{bmatrix} = \begin{bmatrix} J_x \\ J_z \end{bmatrix}. \tag{14}$$

Matrix equation (13) is similar to that written in [4] with different values for elements of the matrix. To find the dispersion characteristics of a slot transmission line, (14) is solved via Galerkin's method.

Results

The results presented here are all for the zero-order solution [4]. Firstly, results were computed for the single enclosed microstrip, as in Itoh and Mittra [4]. Numerical results agreed totally with the earlier figures, as they should, the basic theories being identical for this structure.

The only other theory available for dispersion with multilayer structures is for open structures [5]. To test the new theory, structures were therefore examined with well-spaced conducting walls, to allow some comparison with open structures. Dispersion curves are given in Fig. 2 of two structures, one having identical geometry to that of Knorr and Kuchler [5] except for the enclosing wall. Close agreement is seen between the two theories for $D/b = 0.1$. The closer agreement with increasing frequency

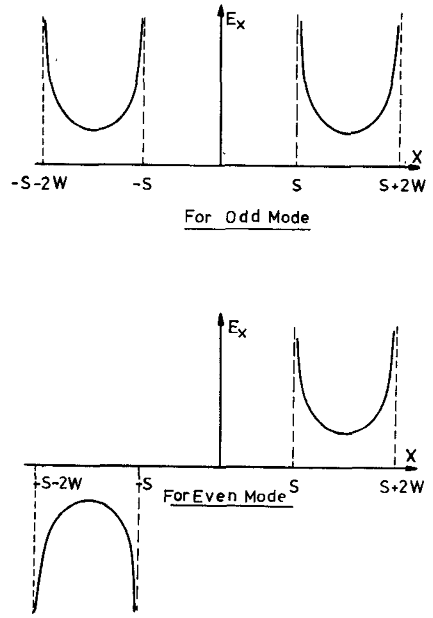

Fig. 3. Distribution of electric field for coupled slots.

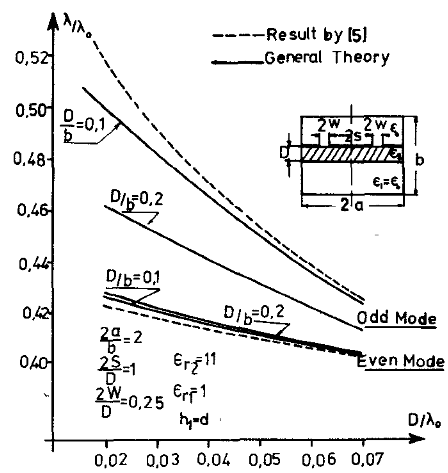

Fig. 4. Even and odd mode dispersion characteristics for coupled slots (even denoting an even E_z, E_y, and H_x).

is consistent with the decreasing effect of the enclosing conductor.

Computing time is about 2 s for each point, and shows the high efficiency of the spectral domain method. There is a clear advantage in the analysis accounting for the enclosing conducting walls, to model the inevitable packaging in practice.

Dispersion characteristics of another type of structure, a shielded coupled slot line, was also investigated. The field distribution within the slots for even and odd modes is shown in Fig. 3, using the dependence $(x^2 - w^2)^{-1/2}$ of [5] and [7]. Fig. 4 shows our results; again a comparison with results of Knorr and Kuchler [5] for open slot lines is given. Computing time for this case is about 6 s per point.

Conclusion

In spite of the increasing interest in suspended substrate transmission line, previous theories have ignored either dispersion, or the presence of the (inevitable) enclosure. The work described here—an extension of the spectral domain approach—allows for efficient computation of slot line, coplanar guide, microstrip, or similar structures (including couplers) with adjacent conductors on the one substrate surface. One computer program deals with an arbitrary number of regions under the coplanar conductors, with simple modifications for different conductors. CPU time on an IBM 360/65 is about 1 s per region for a single slot or strip, at one frequency.

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Application of MIC Formulas to a Class of Integrated-Optics Modulator Analyses: A Simple Transformation

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Abstract—**Modulation electric fields in a class of planar, lumped-parameter circuit or traveling-wave-type electrooptic modulators in integrated optics are analyzed by applying a simple transformation of variables to presently available formulas on microwave integrated circuits (MIC's). Example calculations are shown.**

I. Introduction

It has already been shown that the microwave theory is useful in designing a discrete broad-band electrooptic modulator [1]. Electrooptic modulators in integrated optics are inherently of the planar structure fabricated by microelectronic technology [2]–[6]. While modulators with coplanar electrodes of a narrow gap are expected to have high values of energy efficiency, the nonuniform modulation electric field distribution in the cross section of an optical beam is also apparent.

It is necessary to develop a theory to estimate the electric field distribution in these structures, but such a theory has not been reported. Though there is a structural similarity between such modulators in integrated optics and striplines in microwave integrated circuits (MIC's), the electric field theory of the latter cannot be directly applied to the former since the electrooptic crystal is anisotropic.

This short paper describes a simple transformation of a coordinate and dielectric constants which enables us to apply available formulas and computer programs on MIC's to a class of integrated-optics modulator analyses. Numerical examples based on this method are shown.

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